Obfuscation and Rational Inattention*

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Abstract

We study the behavior of duopolistic firms that can obfuscate their prices before competing on price. Obfuscation affects the rational inattentive consumers' optimal information strategy, which determines the probabilistic demand. Our model advances related models by allowing consumers to update their unrestricted prior beliefs with an informative signal of any form. We show that the game may result in an obfuscation equilibrium with high prices or a transparency equilibrium with low prices and no obfuscation, providing an argument for market regulation. Obfuscation equilibria cease to exist for low information costs and if one firm seems a priori considerably more attractive.

Keywords: Rational Inattention, Obfuscation, Price Competition, Digitalized Markets

JEL Codes: D11, D21, D43

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1 Introduction

Consumers often need to exert costly effort to learn about the price and quality of different products. In many markets, the complexity of price-quality assessment results from firms' obfuscation practices that make it harder for consumers to learn about their offers. Examples of such practices include add-on pricing (e.g., Ellison, 2005; Gabaix and Laibson, 2006), hidden shipping and handling costs (e.g., Hossain and Morgan, 2006; Brown et al., 2010), and complicated or confusing product descriptions (e.g., Ellison and Ellison, 2009; Chioveanu and Zhou, 2013). The optimal degree of obfuscation is a strategic decision by firms subject to consumers' information acquisition process, largely depending on exogenous factors, such as availability of information, information costs, market structure, or consumers' prior beliefs.[1] In recent years, digital technology has reshaped many of these factors (Goldfarb and Tucker, 2019). In particular, it has markedly increased the amount of available and accessible information.

We study equilibrium pricing and obfuscation behavior of profit-maximizing firms in a market with (homogeneous) rational inattentive consumers to account for this development. In contrast to existing models of obfuscation, consumers' information acquisition is endogenous, with no restrictions on consumers' prior beliefs or the type and extent of information processing. We consider a duopoly where firms compete on the price of a homogeneous good. Firms non-cooperatively choose between a transparent pricing scheme and an obfuscated pricing scheme. A transparent pricing scheme implies that consumers can perfectly observe the price set by the firm. Obfuscation implies that consumers do not perfectly observe the price. Consumers can, however, exert costly efforts to decrease uncertainty about obfuscated offers. The optimal information strategy, which depends on consumers' information costs and prior beliefs about the price-quality differentials of the products, determines the endogenous demands for the products. We make the behavioral assumptions that prior beliefs are exogenous, and consumers do not infer equilibrium prices from observing their information costs. Our model has implications for effective market regulation as obfuscation makes it more costly for consumers to learn about firms' offers and, therefore, decreases competition.

The structure of our model is motivated by recent developments in the Swedish and other mobile subscription markets, where pricing schemes have become less complex in recent years. Mobile subscriptions were previously characterized by a complex, variable add-on pricing structure. Today, most subscription plans are subject to fixed, postpaid pricing schemes. A decrease in prices accompanied this development. In our model, this development can arise endogenously due to digitalization, equilibrium pricing, and obfuscation behavior of profit-maximizing firms. In general, our model also applies well to other markets where firms, broadly speaking, can choose between: i) a transparent flat pricing scheme, which makes it easy for consumers to assess products' "final" price per month or year; and ii) a complex variable pricing

[1] Several studies examine various firms' actions that aim to manipulate consumers' information acquisition process in different market settings and with diverging implications for the resulting complexity of price-quality assessment in equilibrium. For example, see Gabaix and Laibson (2006), Carlin (2009), Ellison and Ellison (2009), Ellison and Wolitzky (2012), Chioveanu and Zhou (2013), and Heidhues et al. (2016).

scheme that depends on many details over different dimensions and, therefore, requiring consumers to exert costly efforts to get a reasonable price estimate. Other examples of such markets include different platform services (e.g. social networking sites and online marketplaces), video and online gaming, commercial news websites, and some services in the financial and health industries.

We show that our model may result in an obfuscation equilibrium with high prices and positive profits where both firms obfuscate and a transparency equilibrium with low prices and no obfuscation. The results first highlight that rationally inattentive consumers may incentivize firms in duopolies to optimally obfuscate their prices—even if consumers are homogeneous. If both firms obfuscate their price, market competition is lower as consumers (optimally) only imperfectly learn about obfuscated prices. Obfuscation may be a mutual best reply. This result extends findings from the standard search cost literature (e.g. Ellison and Wolitzky, 2012).

Equilibrium pricing and obfuscation behavior depend on consumers' information costs to learn about obfuscated prices. An equilibrium with high prices and obfuscation only exists if information costs are high enough. Otherwise, the rents from decreased competition in the joint obfuscation may be too low, giving firms an incentive to choose a transparent pricing scheme to secure the entire market. This incentive may arise because consumers optimally do not process any information if the transparent option is very attractive relative to the prior expectation about the obfuscated product. Eventually, this may make obfuscation a dominated strategy, leading to zero equilibrium profits as in Bertrand (1883)—a prisoner's dilemma for the firms.

Additionally, we find that the equilibrium outcome depends on the shape of the consumers' prior beliefs. If one firm seems more attractive according to prior beliefs, i.e., it has higher prominence, the consumer will bias the purchase decisions towards more prominent firms. In a scenario where both firms obfuscate, prices are dispersed if consumers' prior beliefs discriminate between the two firms. A higher prominence implies higher optimal prices, a higher market share, and higher profits. The less prominent firm will claim less of the rent arising from obfuscation, which increases the incentive to use a transparent pricing scheme in an attempt to claim the entire market. Consequently, obfuscation in equilibrium ceases to exist if differences in prominence are large. Similar to Bayesian updaters, rational inattentive consumers optimally put less decision weight on prior beliefs if prior uncertainty is high relative to the variance of the (optimal) informative signal. Consequently, the prominence effect, everything else equal, is less pronounced if the (subjective) prior uncertainty about the distribution of price differences is large and vice versa.

Finally, we find that competition in the form of new market entrants alters optimal obfuscation behavior and prices. First, if the new competitor is significantly less prominent than existing firms, obfuscation equilibria cease to exist. Second, if the new competitor is sufficiently prominent, two channels become important. The competition channel increases competition among obfuscating firms. As a result, existing firms' optimal prices and profits decrease, thereby decreasing the likelihood of obfuscation equilibria. The information channel affects the equilibrium outcome by changing the prior belief and, thus, the consumers' optimal information strategy. This channel may have opposing effects if a new market entry increases the incentive to process at least some information and if the new information strategy favors the less prominent firm. The effect on obfuscation behavior and prices will largely depend on the consumers' prior belief about the new product and its correlation with existing options.

According to these results, digitalization may rationalize the shift towards more transparent pricing schemes in the Swedish mobile subscription market. Digital technologies may have decreased individual information costs to the extent that a transparent pricing scheme has become the dominant firm strategy. The difference to other markets with still complex pricing schemes, such as service contracts in the financial and health industries, could be attributed to lower initial information costs or a comparably larger relative change in information costs due to digitalization.[2] Another possible explanation relates to consumers' prior beliefs. Even if digitalization homogeneously affects information costs, industries with more asymmetric prior beliefs would be less likely to stay in a state of high obfuscation and prices. For example, in the mobile subscription industry, switching costs, industry-specific advertisement and other reasons for habit persistence increase the differences in prominence (Reme et al., 2018). Therefore, decreasing information costs may have larger equilibrium consequences in the mobile subscription market compared to other markets with more symmetric prior beliefs about prices.

Our results further have important implications for market regulation. In addition to direct attention costs for consumers, obfuscation reduces competition among firms and decreases consumer surplus. Consequently, a policymaker may want to regulate markets to avoid obfuscation. According to our model, two main policy options exist. First, there is the option to decrease the information costs for consumers to learn about prices. Lower information costs can be achieved by increasing comparability between prices and facilitating search, for example, by digitalization, or by restricting product features that complicate consumers' assessment of the product's final price, such as add-on pricing. Heidhues et al. (2021) reach a similar conclusion, studying secondary price features of products in a similar setting. Simply providing additional information about the products and prices does not help, as consumers' attention is the limiting factor. Second, there is the option to increase the difference in the prominence of firms, for example, by decreasing market entry barriers for new and unknown competitors that are likely to be less prominent than existing firms. These policies make obfuscation and high prices less likely in equilibrium and increase consumer welfare.

Our demand-side approach takes advantage of a seminal paper by (Matějka and McKay, 2015, henceforth MM15) that formalizes the idea of rational inattentive consumers, initiated by Sims (2003), in a discrete choice framework.[3] Importantly, our framework inherits the idea that consumers have access

[2]For general evidence of inertia in insurance markets, see, for example, Abaluck and Gruber (2016), Handel (2013), Handel and Kolstad (2015), Marquis and Holmer (1996), or Marzilli Ericson (2014). Woodward and Hall (2010) provide evidence for information costs and confusion in the mortgage market.

[3]Caplin et al. (2019) extend this framework by providing necessary and sufficient conditions that allow identification of products about which rational inattentive consumers will process at least some information. For our paper, these conditions

to all information and freely choose what kind of information to process subject to a generally applicable information cost function based on Shannon entropy (e.g., Sims, 2003; Caplin et al., 2019). This makes our approach particularly appealing when relevant information is abundantly available and final prices are complex, depending on common patterns. For instance, in our motivating example, consumers may want to focus their information strategy on learning about their consumption patterns (and not single prices), which will reveal some information about all prices. While existing models are a good fit for window-shopping or browsing explicitly priced products, they can usually not capture more flexible information strategies.

Traditional consumer search models with undifferentiated products (e.g., Stahl, 1989; Ellison and Wolitzky, 2012) usually assume that information about one product is binary. Consumers either do not learn anything about a product or identify the relevant valuation perfectly. Further, consumers can be divided into two types, either "costless" or "costly" searchers. The order of search plays an important role. The models result in a unique symmetric mixed strategy equilibrium, where firms set prices randomly according to distribution that involves a cutoff that, in turn, determines obfuscation behavior. In equilibrium, consumers usually only search once. In contrast to approaches in the search cost literature, rational inattentive consumers make mistakes even if they process information about all products. Even if consumers are homogeneous, obfuscation may be the firms' mutual best reply, and price dispersion is possible in equilibrium if the homogeneous prior belief discriminates between the products. A pure strategy equilibrium exists where firms obfuscate and make positive profits.[4] Rational inattentive consumers typically process information about all obfuscated alternatives in equilibrium.

Related literature

One class of related studies utilize exogenous information structures where a fraction of consumers are sophisticated and make optimal decisions, while another fraction of consumers are naive and choose randomly. In these settings, the firms usually manipulate the relative number of sophisticated consumers by obfuscation. Examples include Gabaix and Laibson (2006), who study add-on pricing; Carlin (2009), who considers the retail financial industry; and Chioveanu and Zhou (2013) and Piccione and Spiegler (2012), who focus on the comparability of prices. Notably, these models do not model the endogenous character of consumers' information search, which has considerable implications for firms' optimal obfuscation and pricing behavior, as well as market regulation.

Other related studies analyze optimal obfuscation behavior in a search-theoretic model along the lines of Salop and Stiglitz (1977), and Stahl (1989). In these (sequential) search models, the information set is endogenous.[5] Ellison and Wolitzky (2012) present a sequential search model in which firms affect the

are particularly relevant for the cases with opposing obfuscation behavior by firms. Huettner et al. (2019) further extend the framework of MM15 for heterogeneous information costs for different choices.

[4]This result depends on the behavioral assumption that consumers take these firms' decisions as given and do not infer equilibrium prices from observing their information costs.

[5]Grubb (2015), Spiegler (2016), and Armstrong (2017) provide overviews of recent relevant theoretical work on the

time needed to assess their prices. In the model of Wilson (2010), firms can influence the consumers' order of the search by obfuscation. Taylor (2017) adopt a search-theoretic model in which obfuscation helps firms to target the most valuable consumers.

The role of prior beliefs is directly related to the literature on the "prominence" of firms (e.g., Armstrong et al., 2009; Armstrong and Zhou, 2011; Rhodes, 2011; Chioveanu, 2019) that deploy similar "binary" search models or exogenous information structures on the demand side. In a related study, Gu and Wenzel (2014) find that more prominent firms always choose maximum obfuscation in a market with either sophisticated or naive consumers. Similarly, Chioveanu (2019) study a duopoly where obfuscation influences the share of naive consumers in the market. Naive consumers choose randomly and are more likely to buy from the more prominent firm, thereby incentivizing the more prominent firm to choose a more complex pricing scheme. In contrast to existing models, prominence affects the equilibrium outcome in our model through consumers' endogenous information acquisition and asymmetric prior beliefs. Predictions of the model may differ as prominence can have large or small effects depending on consumers' information costs and the exact shape of prior beliefs.

The paper by MM15 triggered several recent studies in industrial organization that focus on different aspects, such as inattentive sellers (Matějka, 2015) or inattention to quality and optimal pricing (Martin, 2017).[6] Three studies that incorporate rational inattentive demand in an industrial organization model are particularly relevant. First, Matějka and McKay (2012) investigate equilibria in an oligopoly when consumers are rational inattentive. Second, Matějka (2015) and Boyacı and Akçay (2017) study the optimal pricing behavior of a monopolistic firm that faces a rational inattentive consumer. We add to this literature by extending the models along the obfuscation dimension. This important dimension has a natural connection to rational inattention as the attractiveness of obfuscation practices depends on how and which type of information consumers process (Matějka and McKay, 2012; Grubb, 2015).

2 Model

2.1 Motivation

In Appendix A, we provide a descriptive analysis of the Swedish mobile subscription market to exemplify our motivation. We document a change in the predominant pricing scheme and decreased prices in the last two decades—a development observed in many different countries (European Commission, 2019). Mobile subscriptions were characterized by a variable add-on pricing structure over different dimensions in the past. Today, most subscription plans are based on fixed, post-paid pricing schemes. In 2017, 83% of revenues in the Swedish mobile subscription market could be attributed to fixed fees, compared to 54% in 2011. A price decrease accompanied these changes. For instance, the producer prices for

intersection of industrial organization and consumer search.

[6]See Mackowiak et al. for a detailed review of the budding rational inattention literature in other fields of economics.

telecommunication decreased by around 13% between 2014 and 2020. In comparison, service contracts in the financial or health industry are still based on complex pricing schemes. We present a model in which these diverging developments can be explained by equilibrium pricing and obfuscation behavior of profit-maximizing firms when determining factors of endogenous consumers' information acquisition have been reshaped.

2.2 Model Structure

Firms

There are two firms, indexed by $i = 1, 2$, that produce a homogeneous good of quality $q \in \mathbb{R}$. The firms sell their product to a unit mass of consumers and face two sequential strategic decisions. In $t = 1$, they decide simultaneously on the individual obfuscation parameter $\lambda_i \in \{0, \lambda\}$, i.e., between a transparent pricing scheme ($\lambda_i = 0$) and an obfuscated pricing scheme ($\lambda_i = \lambda$). We focus on binary obfuscation, a simplification for market settings where a simple and complex price frame exists and a firm sticks to one of them. Obfuscation comes at a fixed cost of $\xi > 0$. Obfuscation costs ξ may refer to both direct and indirect costs.[7] We discuss the implications of relaxing the assumption of positive obfuscation costs in section 3.3. For obfuscated pricing schemes, consumers need to gather costly information, as described in the following section. Obfuscated prices do not necessarily imply that consumers do not know anything about prices. After observing the obfuscation choice of the other firm, firms simultaneously set their prices ($p_i \geq 0$) in a subsequent second step ($t = 2$). Firms decide about obfuscation as well as prices non-cooperatively and with complete information. We chose a sequential game setup as it is easy to understand, and we believe that obfuscation decisions are rather long-term decisions while prices could vary in the short term. Yet, the results of our model do not depend on the sequential game setup. We consider a simultaneous move game in Appendix C, which yield identical results.

The maximization problem of firm i in the second stage is given by:

$$\max_{p_i \geq 0} \eta_i(p_i, p_{-i}, \lambda_i, \lambda_{-i}, G_0(\cdot)) * p_i - \mathbf{1}_{\lambda_i = \lambda} * \xi, \tag{1}$$

where η_i represents the expected market share or, equivalently, the conditional probability that consumers buy the product of firm i at prices p_i and p_{-i}. The conditional probabilities are a result of the consumers' optimal information strategy (see section 3.2.), which depends on the firms' obfuscation choices λ_i and λ_{-i} as well as the consumers' prior beliefs about prices, $G_0(\cdot)$.

[7] Examples of direct costs include increased costs for labeling the same product differently or (online) marketing expenses targeted at making firms' offers less transparent or comparable to other products. Indirect cost may, for instance, arise because complex pricing schemes may increase administrative efforts necessary to keep track of diverging individual consumption and contracts or lead to higher demand for customer services, dealing with complaints and questions. Ensuring a transparent pricing scheme can also be costly. Costs for a transparent pricing scheme could arise due to advertising or marketing the transparent price to consumers. Obfuscation costs in our model can be defined as relative to fixed costs of a transparent pricing scheme (normalized to zero). Accordingly, positive obfuscation costs generally imply that an obfuscated pricing scheme is more costly than a transparent pricing scheme.

Consumers

Consumers can either buy from firm 1 or firm 2. The payoff of buying product i is given by the price-quality differential: $k_i = q - p_i$. We suppose consumers are aware of quality q, but do not observe the prices set by firms. They have an exogenously given prior belief about the joint distribution of prices, which we denote in terms of the quality-price differentials k_i: $G_0(k_1, k_2) = G_0(\mathbf{k})$. Note that the prior beliefs are defined jointly over each firm's price. In section 4, we exemplify how larger competition could affect equilibria through prior beliefs. The prior belief fulfills two main functions in our model.

First, it initiates uncertainty about prices. We do not restrict consumers' prior beliefs to allow for different rationales behind the uncertainty in our model. Accordingly, consumers' may not have rational expectations about prices. Prior beliefs can refer to (a combination of) many different factors, including uncertain future consumption, unintelligible firm structures, production costs, and other behavioral biases and misunderstandings.[8] Second, prior beliefs determine how prominent the firms are relative to each other. We consider a firm to be more prominent if consumers expect that the respective firm sets a lower relative price according to their prior beliefs. Similar qualitative results would arise if we assume that different levels of prominence result from diverging prior beliefs regarding the relative qualities or marginal costs of the products.

The consumers have the possibility to gather and process signals $\mathbf{z}$ to decrease the uncertainty about prices set by firms. The information strategy is unrestricted and can be denoted as a joint distribution of signals and payoffs, $F(\mathbf{z}, \mathbf{k}) \in \Delta(\mathbb{R}^2)$. $\Delta(\mathbb{R}^2)$ denotes the set of all probability distributions on $\mathbb{R}^2$ for both firms. The information strategy has to be consistent with the consumers' prior belief. The condition $\int_z F(dz, \mathbf{k}) = G_0(\mathbf{k})$ assures this. Receiving signals $\mathbf{z}$, according to the respective information strategy, results in a posterior belief, $F(\mathbf{k}|\mathbf{z})$. Given this posterior belief, the consumers choose product a with the highest expected payoff, i.e. the lowest expected price:

$$a(F(\mathbf{k}|\mathbf{z})) = \arg\max_i E_{F(\mathbf{k}|\mathbf{z})}(q - p_i)$$

When choosing the optimal information strategy, the consumers weigh the cost of information processing against its expected benefits. The benefits arise because choosing a distribution of more precise signals results in less uncertain posterior beliefs, which translates into a higher expected value of the optimal choice. More precise signals are, however, more costly. We follow the rational inattention literature and assume an information cost function that is linear in the expected Shannon entropy reduction by means of the processed signal: $\hat{c}(F) = \lambda(H(G_0) - E_z[H(F(\mathbf{k}|\mathbf{z})]))$. $H(B)$ denotes the Shannon entropy of distribution B. λ is the unit cost of information, which is equivalent in all cases with at least one

[8] To make prices directly related to the equilibrium distribution of prices and satisfy rational expectations, one could, similar to Matějka (2015), assume that consumers are aware of the optimal pricing strategies of firms but imperfectly observe the realization of firms' marginal costs.

obfuscated price.[9] Consumers observe their information costs. We assume that consumers do not infer firms' obfuscation strategies and equilibrium prices from their information costs. λ can be best interpreted as consumers' personal costs of processing price information, depending on factors that cannot be directly influenced by firms, such as available time, experts at hand, access to digital technology, etc.

While the number of obfuscating firms does not influence λ, it influences the prior beliefs relevant to the optimal information strategy. If both firms obfuscate, we deal with a joint (multivariate) probability distribution about both quality-price differentials, $G_0(\mathbf{k})$. If one firm chooses a transparent pricing option, the price of the transparent product is perfectly observable, and the optimal information strategy depends on the prior belief conditional on transparent price-quality differentials $\tilde{k}$, $G_0(\mathbf{k}|\tilde{k})$. In the duopoly case, $G_0(\mathbf{k}|\tilde{k})$ is a univariate probability distribution. The entropy of this conditional distribution, and thus the uncertainty about the best option, is typically smaller than in the joint obfuscation case. Consequently, rational inattentive consumers tend to process more costly information in the joint obfuscation case. Yet, the resulting posterior beliefs are typically more uncertain, and consumers make a less informed decision, implying more mistakes in the joint obfuscation case.

Formally, the consumers choose an information strategy to solve the following maximization problem:

$$\max_{F\in\Delta(\mathbb{R}^{2N})} \int_{\mathbf{k}}\int_{\mathbf{z}} k_{a(F(\mathbf{k}|\mathbf{z}))} F(d\mathbf{z}|\mathbf{k}) G_0(d\mathbf{k}) - \hat{c}(F)$$
$$\text{s.t. } \int_z F(d\mathbf{z},\mathbf{k}) = G_0(\mathbf{k})$$

3 Equilibrium

3.1 Characterization

We focus on subgame perfect equilibria in pure strategies. S is the set of strategy profiles, i.e., the set of possible pairs of firms' strategies, $(s_1, s_2) \in S$. Firm i's strategy $(s_i = (\lambda_i, p_i(\lambda_i, \lambda_{-i}))$ consists of the obfuscation decision in the first stage (λ_i) and the pricing decision in the second stage conditional on firms' obfuscation choices $(p_i(\lambda_i, \lambda_{-i}))$, taking into account the optimal information strategy of the rational inattentive consumers. A strategy profile $s \in S$ is subgame perfect if for each subgame the strategy profile constitutes a Nash equilibrium. A strategy profile $s \in S$ is a pure Nash equilibrium if $\forall i$ and $\forall s_i \in S_i$, $U_i(s_i, s_{-i}) \geq U_i(s_i', s_{-i})$.

[9]This notation is possible since we focus on binary obfuscation choices, i.e., we restrict the individual information costs to zero and λ. If the firms can set individual non-zero information costs different from each other (continuous obfuscation choice), $0 < \lambda_1 < \lambda_2 < \infty$, the problem gets more pronounced (see Huettner et al., 2019) and the results of the model may change.

3.2 Consumer Demand

If both firms choose a transparent pricing scheme ($\lambda_1 = \lambda_2 = 0$), the information strategy and the derivation of the consumer demands are trivial. The consumers will choose (costless) signals that perfectly identify firms' prices, implying that our model becomes deterministic. Consumers will buy the product with the lowest price. The conditional choice probabilities $\eta_i(\mathbf{k})$, which signify the probability that consumers choose product i at $k_i = q - p_i$, $\forall i \in 2$, will thus equal 1 if $p_i < p_j, \forall j \neq i$ and zero otherwise. If two or more actions exhibit the same state-contingent payout, a tie-breaking rule is applied, resulting in equally distributed conditional choice probabilities among these options.

If at least one firm chooses to obfuscate, information processing is costly, implying probabilistic demands and posterior uncertainty about prices set by firms. In these cases, one essential and well-known feature of rational inattention is that each action strategy is associated with a particular signal (MM15), which allows us to rewrite the formal maximization problem of the consumers in terms of choice probabilities. Solving the reformulated consumers' maximization problem, we show, following MM15, that the optimal conditional choice probabilities $\eta_i(\mathbf{k})$ will follow a multinomial logit function (See Appendix B):

$$\eta_i(\mathbf{k}) = \begin{cases} 0 & \text{if } \eta_i^0 = 0 \\ \dfrac{\eta_i^0 e^{(q-p_i)/\lambda}}{\eta_1^0 e^{(q-p_i)/\lambda} + \eta_2^0 e^{(q-p_{-i})/\lambda}} & \text{if } 0 < \eta_i^0 < 1, \\ 1 & \text{if } \eta_i^0 = 1 \end{cases} \tag{2}$$

where η_i^0 are the unconditional choice probabilities, which are the collection of all conditional choice probabilities $\eta_i(\mathbf{k})$ integrated over the respective prior beliefs $G_0(\mathbf{k})$: $\eta_i^0 \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) G_0(d\mathbf{k})$. Accordingly, the unconditional choice probabilities result from the consumers' maximization problem and will shift the demand towards the more prominent product. η_i^0 is independent of the prices set by firms but affected by the prior belief and λ. As a result, even large differences in prominence may only marginally affect conditional choice probabilities if λ is small or prior beliefs are very uncertain. Intuitively, prominence has a small effect if prior beliefs are unimportant relative to the informative signal $\mathbf{z}$. Similar to standard Bayesian updating, this can happen either because the received signal will be very informative due to small information costs or because the prior beliefs are not precise. If consumers do not distinguish between the different products according to their prior beliefs, both unconditional choice probabilities will equal $\eta_i^0 = 0.5$. In this case, the model reduces to a symmetric logit model, and both products are completely homogeneous.

The consumers' optimal probabilistic choice will follow a multinomial logit form, irrespective of whether only one firm chooses to obfuscate or both firms choose to obfuscate (See Appendix B for derivations). However, the cases differ concerning the relevant (conditional) prior beliefs because rational inattention implies that consumers process all costless information, i.e., the transparent price, before choosing the optimal (non-trivial) information strategy. For the case in which only one firm obfuscates, this implies that the prior belief conditional on all costless information $\tilde{k}$, $G_0(\mathbf{k}|\tilde{k})$ is relevant for the consumers' maximization problem. We assume that, according to the consumers' prior belief, the price of one firm is independent of the price of the other firm such that learning the price of the transparent firm does not imply anything about the other price. As a result, the corresponding unconditional choice probabilities η_i^0 differ, which has important implications, particularly concerning border cases where the consumers do not process any information. Unconditional choice probabilities of zero and one signify these border cases. $\eta_i^0 = 1$ implies that the consumer would not process any information and buys product i with a probability of one ($\eta_i(\mathbf{k}) = 1$) irrespective of the prices set by obfuscating firm(s). Accordingly, the consumers never choose or consider a product with $\eta_i^0 = 0$. Caplin et al. (2019) provide the corresponding necessary and sufficient conditions for the unconditional choice probabilities that we apply in our simulations later. If $0 < \eta_i^0 < 1$, the normalization conditions laid out by MM15 apply.

The Price Cutoff Level

Caplin et al. (2019) show that rational inattentive consumers may never consider some options in a discrete choice setting. These non-considerations of options arise endogenously and will depend on a unique cutoff that depends on the information cost and prior beliefs (Caplin et al., 2019). In our setting, non-considerations of a product exist if information costs are comparably high and one of the products seems a priori much more attractive to the consumer than the other option. While we assume that the prior beliefs are such that consumers always find it optimal to consider both products if both firms choose to obfuscate, the cutoff level becomes essential in the case of opposing obfuscation choices by firms.

If the price of the transparent firm, which is costless information to the consumers, is low enough, the consumers will not process any information. They will buy from the transparent firm with a probability of one. We denote the cutoff price level as $\bar{p}_i$ ($\lambda_i = 0$ and $\lambda_{-i} = \lambda$): $\bar{p}_i = \max p_i$ such that $\eta_i^0 = \eta_i(\mathbf{k}) = 1$. If the transparent price is below that cutoff, the agent will not process any information and will always buy the transparent product. If the transparent price is above the cutoff, the consumers' likelihood of buying the obfuscated product will be positive. While the exact cutoff level will depend on the shape of the prior belief and the information costs (see simulations below), some properties are important for the game's outcome.

First, the cutoff level will always be larger than zero for positive information costs and some prior uncertainty about the obfuscated price, which implies there will also be some remaining posterior uncertainty about the obfuscated price after processing information. Consider a transparent price that is set to or close to zero. As we assume positive prices this implies that even in the most favorable (uncertain) state, the transparent option is as good as the obfuscated option. As a result, consumers would never process any information and buy the transparent product if the transparent price is close enough to zero.

Second, the price cutoff level is increasing in λ. Under rational inattention, the obfuscated product is *a priori* more valuable for the consumers as the prices vary in every state (from the perspective of the consumers). Consumers can tailor the optimal information strategy to purchase the transparent product if the obfuscated price is high and the obfuscated product otherwise. Expected variance becomes beneficial because the consumers can customize their choice to the realization of the state, which, in our case, refers to the prices set by the obfuscating firm (see also Caplin et al., 2019). If information costs increase, it gets more costly to customize the choice. The advantage of the obfuscated option vanishes, and the consumer will choose the transparent option at larger prices without processing any information, implying an increasing cutoff level.

Third, the cutoff level converges to the expected price of the obfuscated product if information costs strive to infinity. If $\lambda = \infty$, the consumer will not process any information and will always buy the obfuscated product if its expected price at prior beliefs is lower than the transparent price and vice versa. Accordingly, the expected price at prior beliefs constitutes an asymptote for the price cutoff level for increasing levels of λ.

3.3 Firms

Second Stage

In the spirit of backward induction, we start by solving for subgame perfect equilibria of the competitive pricing game of the firms conditional on the obfuscation decision made in the first stage.

Proposition 1. *If both firms choose to obfuscate, the optimal prices and profits are uniquely determined by following set of equations:*

$$p_i^* = \frac{\lambda}{(1 - \eta_i^*(\boldsymbol{k}))}. \tag{3}$$

for $i \in N$, where $\eta_i^*(\boldsymbol{k})$ denote the equilibrium market share, and the corresponding equilibrium profits of firm i are given by:

$$\pi_i^* = \frac{\lambda}{(1 - \eta_i^*(\boldsymbol{k}))}\eta_i^*(\boldsymbol{k}) - \xi \tag{4}$$

Proof. Given the multinomial logit demand function derived above, the derivation of equilibrium prices and profits in the case of simultaneous obfuscation follows a similar logic as in Anderson et al. (1992).[10] The first-order conditions are given as follows:

$$\eta_i(\mathbf{k}) - \frac{\eta_i(\mathbf{k})(1 - \eta_i(\mathbf{k}))}{\lambda}p_i^* = 0,$$

For an interior solution, the optimal price of firm i is thus given by equation 3. Anderson et al. (1992) show that such a system of equations has a unique solution. Plugging the equilibrium prices back into

[10] In Anderson et al. (1992) the multinomial logit demand arises from a representative consumer with linear random utility and double exponential distributed error terms. In contrast, under rational inattention, the randomness arises not from noise with respect to preferences, but from the signal the consumer receives.

the profit function (equation 1), gives us the equilibrium profits (equation 4). □

As noted above, the unconditional choice probabilities are independent of firms' prices as they do not affect the consumer's prior belief. As a result, firms cannot influence the unconditional choice probabilities in the second stage if $\lambda_i = \lambda$. If only one firm chooses to obfuscate, the consumer will be perfectly informed about the transparent price set in the second stage. Consequently, the prior belief conditional on all costless information ($G_0(\mathbf{k}|\tilde{k})$) changes, which affects the consumer's optimal information strategy. As a result, the transparent firm can influence the unconditional choice probabilities. See Appendix B for further descriptions.

Proposition 2. *If only one firm chooses to obfuscate, the optimal price of the transparent firm ($\lambda_i = 0$) is equal to $\bar{p}_i$, where $\bar{p}_i = \max p_i$ such that $\eta_i^0 = \eta_i(\boldsymbol{k}) = 1$.*

Proof. First, note that the transparent firm will not set a price below $\bar{p}_i$, because, if $p_i < \bar{p}_i$, $\eta_i(\mathbf{k}) = 1$ and accordingly $\dfrac{\partial \pi_i}{\partial p_i} = 1 > 0$.

If $p_i \geq \bar{p}_i$, both firms will face a multinomial demand as derived above. The maximized profit function for a given unconditional choice probability is equal to $\pi_i^*(\eta_i^0) = \dfrac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k})$. Plugging in the corresponding demand function, the profit function can be rewritten to:

$$\pi_i^*(\eta_i^0) = \lambda \frac{\eta_i^0}{(1-\eta_i^0)} \frac{e^{(q-p_i^*)/\lambda}}{e^{(q-p_{-i}^*)/\lambda}} = \lambda \frac{\eta_i^0}{(1-\eta_i^0)} e^{(p_{-i}^* - p_i^*)/\lambda}$$

Applying the Envelope theorem, the derivative of the profit function with respect to the unconditional choice probability is given by:

$$\frac{\partial \pi_i^*}{\partial \eta_i^0} = \frac{\lambda}{(1-\eta_i^0)^2} e^{(p_{-i}^* - p_i^*)/\lambda} > 0,$$

which is strictly larger than zero if $\eta_i^0 < 1$.

□

As a consequence of Proposition 2, the transparent firm's profits and optimal price in equilibrium are given by $\pi_i = p_i^* = \bar{p}_i$. The obfuscating firm has a zero market share and can set any p_2 in equilibrium. The equilibrium profits of the obfuscating firm are equal to $-\xi$. Proposition 2 further implies that the consumer will not process any information. The exact value of the cutoff level $\bar{p}_i$ depends on the prior beliefs, and the information costs parameter λ, as we discuss in the sections below.

If both firms choose not to obfuscate, the pricing game is equivalent to a well-studied Bertrand duopoly. Both obfuscation parameters are equal to zero such that the rational inattentive consumer observes prices perfectly. The unique best reply of both firms is given by $p_1 = p_2 = 0$, the Bertrand paradox. The corresponding firm profits are zero.

First Stage

In the first stage, both firms have the option to obfuscate or choose a transparent pricing structure. The simultaneous and discrete decision to obfuscate leads to four different nodes of the game and three different possible equilibria. Following the results of the second stage, we can represent the simultaneous move game in the first stage in a symmetric 2×2 payoff matrix presented in Table 1.

Table 1: First Stage Payoff matrix

		Firm 2	
		$\lambda_2 = 0$	$\lambda_2 = \lambda$
Firm 1	$\lambda_1 = 0$	$(0, 0)$	$(\bar{p}_1(G_0(\cdot), \lambda), -\xi)$
	$\lambda_1 = \lambda$	$(-\xi, \bar{p}_2(G_0(\cdot), \lambda))$	$\left(\frac{\lambda}{(1-\eta_1^*(\mathbf{k}))}\eta_1^*(\mathbf{k}) - \xi, \frac{\lambda}{(1-\eta_2^*(\mathbf{k}))}\eta_2^*(\mathbf{k}) - \xi\right)$

Notes: The table presents the payoff matrix of both firms in the first stage. Firms 1 and 2 set their obfuscation parameters simultaneously. Each combination of obfuscation parameters relates to a payoff function.

Proposition 3. ***Subgame perfect equilibria:***

i.) ***Transparency equilibrium:*** *There always exists a subgame perfect equilibrium that is defined by the following strategies:*

$$s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$$

ii.) ***Opposing equilibrium:*** *There is no opposing equilibrium in which only one firm obfuscates.*

iii.) ***Obfuscation equilibrium:*** *There exists a subgame perfect equilibrium that is defined by the following strategies and conditions:*

Strategies:

$$s_i = (\lambda_i = \lambda, p_i = c + \frac{\lambda}{(1-\eta_i^*(\boldsymbol{k}))}) \quad \forall i \in N$$

Condition:

$$\frac{\lambda}{(1-\eta_i^*(\boldsymbol{k}))}\eta_i^*(\boldsymbol{k}) - \xi \geq \bar{p}_i(G_0(\cdot)) \quad \forall i \in N \tag{5}$$

Proof. If both firms choose a transparent pricing scheme, prices and profits are zero. In the case that the opponent of a firm does not obfuscate, i.e., $\lambda_{-i} = 0$ where $-i \in N \setminus i$, it is always the best reply for firm i to be transparent ($\lambda_i = 0$) as $0 > -\xi$. Thus, the Transparency equilibrium $s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$ always exists independent of prior beliefs and information costs if $\xi > 0$ (*Proposition 3.i.*).

Following Proposition 2, the transparent firm claims the entire market if $\xi > 0$ and $\lambda_i \neq \lambda_{-i}$. As a result, only the transparent firm would make non-negative profits, while the obfuscating firm would suffer a loss due to positive obfuscation costs. Consequently, it is the best reply to be transparent as well, yielding a payoff of zero. An opposing equilibrium does not exist (*Proposition 3.ii.*).

According to *Proposition 3.iii.)*, the game has an obfuscation equilibrium ($s_i = (\lambda_i = \lambda, p_i = \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))})$) if condition (4) holds. Condition (4) states that firm i's profit in the joint obfuscation case is larger than the profit that firm i would gain with a transparent pricing scheme, given the other firm chooses to obfuscate. As shown above, the profit of firm i in the joint obfuscation case is given by: $\frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k}) - \xi$ (Proposition 1); and the profit of the transparent firm in the opposing case is equal to the cutoff price level $\bar{p}_i(G_0(\cdot))$ (Proposition 2), which may be different for each firm (see below). If condition (4) does not hold, firm i would have an incentive to deviate from choosing obfuscation in the first stage and the obfuscation equilibrium cease to exist. □

Overall, condition (4) is crucial for the game's outcome in the first stage. The first stage is a pure coordination game with two equilibria if the condition holds. The payoff is strictly higher in the obfuscation equilibrium than in the transparency equilibrium. If the condition does not hold, the game is equivalent to a prisoner's dilemma. Even though mutual obfuscation would result in a higher payoff, the game has a unique equilibrium in pure strategies, in which both firms set $\lambda_i = 0 \; \forall i \in N$.

Furthermore, condition (4) also illustrates the role of obfuscation costs ξ. While ξ does not affect optimal prices in the second stage, it has important implications for the outcome of the first stage. We assume positive obfuscation costs, referring to direct and indirect costs (see examples above). In most relevant markets, this seems like a realistic assumption. However, there might be situations where obfuscation costs may be zero or even negative, for example, if costs for ensuring a transparent pricing scheme are very high. For $\xi \leq 0$, the equilibrium outcome can change. Generally, it makes obfuscation more attractive, implying that obfuscation equilibria exist for a larger range of parameters. If $\xi \leq 0$, "opposing" equilibria may exist as both options in the first stage will at least yield a profit of zero, given that the other firm chooses a transparent pricing scheme.[11] For strictly negative obfuscation costs, transparency equilibria cease to exist. Furthermore, if ξ close enough to zero, obfuscation may become the dominant strategy for both firms. Additionally, note that "opposing equilibrium" may exist if we relax the assumption of binary obfuscation choices.

4 The Existence of Obfuscation in Equilibrium

We have shown that two equilibria may exist in the first stage. First, there always exists a transparency equilibrium, in which both firms do not obfuscate. This equilibrium is independent of λ and consumers' prior beliefs. Second, there may exist an obfuscation equilibrium in which both firms obfuscate. The condition for the existence of the obfuscation equilibrium is described in Proposition 3. The existence depends on the prior beliefs and λ. While there is no closed-form solution for the existence condition, we

[11] Note that

provide comparative static exercises in this section to demonstrate the effects of information costs and prior beliefs on the existence of an obfuscation equilibrium. In the symmetric benchmark case, we assume that prior beliefs follow a multivariate normal distribution $G_0(\mathbf{k}) \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ with $\boldsymbol{\mu} = (\mu_1, \mu_2) = (4, 4)$ and $\boldsymbol{\Sigma} = (2, 0; 0, 2)$. Accordingly, the difference in prominence, which is defined as the expected price difference according to the prior beliefs, $(\mu_2 - \mu_1)$, is equal to zero. [12]

The Effect of Information Cost Parameter λ

Figure 1 illustrates firms' profits for different obfuscation decisions and different values of λ in the benchmark case. Symmetric prior beliefs imply that consumers do not distinguish between the different products before processing information. The unconditional choice probabilities will be identical across firms, $\eta_i^0 = 0.5$, making them completely homogeneous. When both firms obfuscate, the optimal prices are given by $p_i^* = 2\lambda$ with corresponding profits of $\pi_i = \lambda - \xi$ (solid line). Accordingly, the equilibrium profits and prices linearly increase in λ. Both firms benefit from increasing information costs because this decreases competition among firms. If both firms do not obfuscate, profits are always zero for both firms (connected dots). If only one firm chooses to obfuscate, the obfuscating firm has a negative profit of $-\xi$ (dotted line), while the transparent firm earns a profit that mirrors the cutoff price level $\pi_i = \bar{p}_i$ (dash-dotted line). Importantly, an obfuscation equilibrium only exists, i.e., the condition in Proposition 3 holds, if the profit in the mutual obfuscation case (solid line) is larger than $\bar{p}_i$.

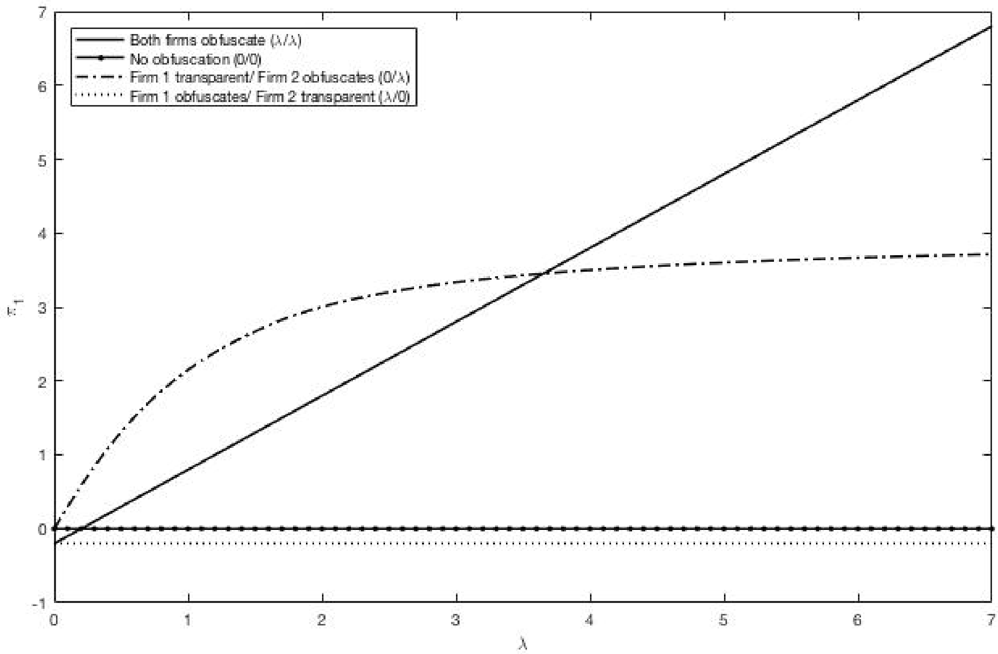
Figure 1 shows that obfuscating the price is a dominated strategy for small values of λ in our simulation exercise. Independent of the opponent's decision, it is optimal for a firm not to obfuscate. The unique subgame perfect equilibrium is that both firms abstain from obfuscation in the first stage and set prices equal to zero in the second stage. The first-stage game reduces to a prisoner's dilemma as both firms could increase their profits if both obfuscate.

For large values of λ, it may become optimal to obfuscate prices, and the game may result in an obfuscation equilibrium. In Figure 1 we observe that for large values of λ, conditional on an obfuscating opponent, the profit of mutual obfuscation exceeds the profit of being transparent. Thus, for symmetric priors and high enough values of λ, there exists a subgame perfect equilibrium where both firms obfuscate and set positive prices, as described in Proposition 3. As there are two equilibria, the game is characterized by a coordination problem. Both firms have an incentive to coordinate to attain the high-payoff equilibrium with positive profits.

The primary rationale behind this finding lies in the price cutoff level. As argued in section 3.2, $\bar{p}_i$ is increasing in λ, always larger than zero, and converges to the expected price of the obfuscated product for larger values of λ. These characteristics of $\bar{p}_i$ imply positive and decreasing marginal profits with respect to λ for the transparent firm in the case of opposing obfuscation decisions (dash-dotted line in Figure 1).

[12] Consequently, the difference in prominence follows following normal distribution $\sim N(\mu_2 - \mu_1, \sigma_{11}^2 + \sigma_{22}^2)$ in the case where both firms obfuscate, $(p_2 - p_1) \sim N(\mu_2 - p_1, \sigma_{22}^2)$ if only firm 2 obfuscates, and $(p_2 - p_1) \sim N(p_2 - \mu_1, \sigma_{11}^2)$ if only firm 1 obfuscates.

Figure 1: Profit Functions for Symmetric Prior Beliefs



Notes: The graph describes the profits of firm 1 for increasing obfuscation (λ) and different obfuscation choices when consumers have symmetric prior beliefs across all prices. The graph for firm 2 is identical.

Furthermore, profits are always non-negative for the transparent firm, implying that for small enough λ and $\xi > 0$, deviating from mutual obfuscation is beneficial as a transparent pricing scheme would yield higher profits conditional on an obfuscating opponent. At the same time, the decreasing marginal profits imply that for high enough values of λ, mutual obfuscation becomes the best reply for both firms, given that marginal profits in the mutual obfuscation case are constant in λ. This result may change in the asymmetric case where profits are not linear in λ.

The Effect of Different Levels of Prominence

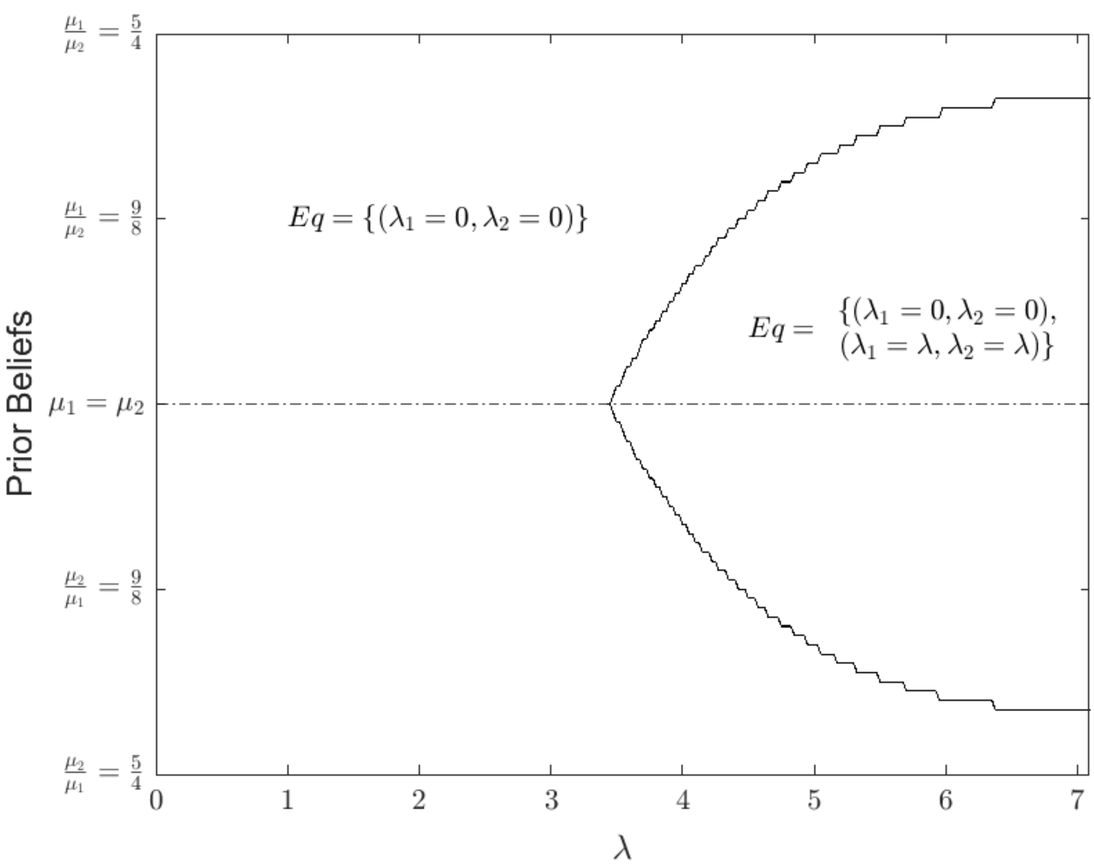
Next, we turn to analyze equilibria under asymmetric prior beliefs of consumers, i.e., diverging prominence of firms. Now, the degree of asymmetry in prior beliefs and the size of information costs determine if the game has one or two subgame perfect equilibria.

Figure 2 shows the relationship between consumers' prior beliefs, information costs, and the existence of an obfuscation equilibrium. In detail, we show the area in which condition (4) holds, i.e., an obfuscation equilibrium exists. The symmetric case refers to the horizontal line ($\mu_1 = \mu_2$). We observe that a higher (absolute) difference in prominence, $|\mu_1 - \mu_2|$, requires a higher λ such that mutual obfuscation in the first stage is an equilibrium, everything else being equal. The intuition can be derived from Figure 3.

Suppose consumers prefer firm 2 to firm 1 before processing information.[13] The difference in promi-

[13] In our simulation for Figure 3, we assume that $G_0(p_1) \sim N(4.4, 2)$ and $G_0(p_2) \sim N(4, 2)$, implying that, before processing information, the consumer expects the product of firm 1 to be more expensive, i.e., firm 2 seems more attractive.

Figure 2: Equilibria in First Stage



Notes: The graph describes equilibria regions for the first stage of the game given different prior beliefs and increasing values of the obfuscation parameter λ. The solid line represents the border between the equilibria regions.

nence translates into different unconditional choice probabilities. For all values of λ, market shares, i.e., conditional choice probabilities, and optimal prices are larger for the more prominent firm if both firms obfuscate. In Figure 3b, we show both firms' profit functions for increasing values of information costs λ, differentiating between the case in which both firms obfuscate and the case in which only one firm obfuscates. If information processing is more costly, the rational inattentive consumer will optimally choose less precise signals. As in the symmetric case, the competition among firms decreases, which benefits both firms. Consequently, the firms' profits are larger for higher values of λ in the mutual obfuscation case (solid lines). However, processing less precise signals also imply that the consumers will rely more on prior beliefs when making their decisions. Differences in prominence, thus, play a more important role. As a result, the gap in unconditional choice probabilities (see Figure 3a) and profits between both firms (see Figure 3b) also widens for higher information costs (λ).

As in the symmetric case, the obfuscating firm makes a negative profit of $-\xi$ if the other firm chooses a transparent pricing scheme. The transparent firm will set $\bar{p_i}$ and has a market share of one. However, the difference for the asymmetric case is that the relevant expected price, constituting the asymptote for profits of the transparent firm, varies conditional on which firm obfuscates. The consumer finds it less beneficial to learn something about an obfuscated price if the consumer expects the price to be comparably high. This implies that the more prominent firm, as compared to the less prominent firm, can set a higher $\bar{p_i}$, yielding higher profits, for the same values of λ.

Proposition 3 requires that the profit from mutual obfuscation exceeds the profit in the opposing case for each firm. Comparing the profit functions for asymmetric priors to those with symmetric priors in Figure 1, we see that the less prominent firm requires a higher value of λ, conditional on the other firm choosing to obfuscate, to prefer obfuscation compared to a transparent pricing scheme. Intuitively, while the benefit of decreased competition persists for both firms, the market share and relative profits for the less prominent firm in the mutual obfuscation case will be lower. This, in turn, increases the relative attractiveness of choosing a transparent pricing scheme. Accordingly, for some values of λ where obfuscation would have been the best reply in the symmetric case, the ex-ante less preferred firm now chooses a transparent pricing scheme such that mutual obfuscation is not an equilibrium anymore.

In our model, the transformed unconditional choice probabilities ($\alpha_i = \lambda log(\eta_i^0)$) plus firm's quality q_i have the same effect as product quality in the standard logit-demand model derived from a random utility model (Anderson et al., 1992; Matějka and McKay, 2012). Consequently, if firms sell products of different qualities, significant differences in firms' qualities—similar to differences in prominence—could prevent obfuscation in equilibrium. To see this, suppose that before processing information, consumers do not distinguish between the two options (despite different qualities), prior beliefs about prices are independent of qualities, and quality levels are exogenous. In this case, it is straightforward to see that a higher-quality firm charges higher prices and makes higher profits in equilibrium, yielding the same equilibrium implications for firms' obfuscation behavior. If firms choose quality strategically—in addition to prices—or when prior beliefs depend on observed quality, the problem gets considerably more involved. We leave a detailed analysis of the effect of quality choice on the attainability of obfuscation for future research.

The Effect of Prior Uncertainty

The consumer demand depends not only on the mean of the prior beliefs but also on their variance (and potentially on other higher-order risk moments). Similar to Bayesian updating, a smaller (subjective) prior uncertainty about firms' prices implies a larger decision weight on prior beliefs. Intuitively, a low uncertainty about prices implies that the consumers (potentially wrongly) expect their priors to represent the true distribution of prices more accurately. The expected value of additional (costly) information on actual price differences decreases. The relative importance of prior beliefs in making a decision increases, which has two main implications for the equilibria in our model.[14]

First, a lower variance of the prior beliefs about both prices (σ^2_{ii}, $i = 1, 2$) implies that the importance of prominence increases in the mutual obfuscation case. Accordingly, the difference between the unconditional choice probabilities (see Figure 3c) and thus profits (Figure 3d) become larger for the same values of λ. Everything else equal, lower variance in our model thus implies that the less prominent firm will deviate from the mutual obfuscation case already for lower values of λ. Thus, obfuscation equilibria cease to exist for lower values of λ and constant prominence, as illustrated by the shifted intersection of the blue lines in Figure 3d.

Second, the price cutoff level ($\bar{p}_i$) and, thus, the profit of the transparent firm's profits in the opposing case are higher if the variance of the obfuscated price is smaller. Intuitively, the expected value of additional information about the obfuscated price decreases if its expected variance decreases. Consequently, the price cutoff level $\bar{p}_i$ increases for the same values of λ and constant means. Again, the area where obfuscation equilibria exist shrinks. The more pronounced convergence to the expected mean of the obfuscated price for smaller values of λ illustrates this effect (compare dash-dotted lines between Figure 3b and 3d).

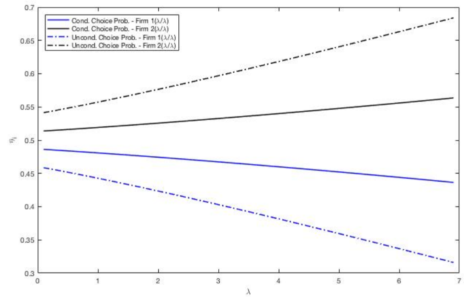
New competitor

We now consider the entry of a new competitor. Our analysis focuses on how equilibrium behavior in our model shifts if a third firm enters the duopoly market before the two incumbents commit to their obfuscation strategy. The new competitor is identical to the incumbents but may differ in prominence. The timing of the game is the same as in the duopoly case. Suppose the model input factors are such that the market would be in an obfuscation equilibrium without a new competitor. There are two potential scenarios.

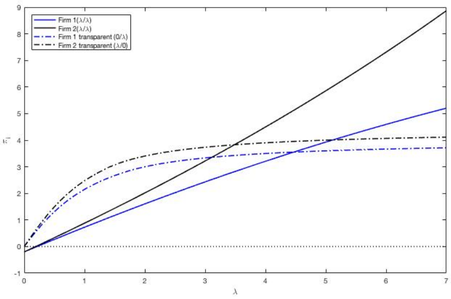
[14] If we relax assumptions of non-zero covariances between expected product' prices and unskewed priors, the effect of the variance may change. A detailed discussion of these effects go beyond the scope of this paper.

Figure 3: Choice Probabilities and Profit Functions for Asymmetric Prior Beliefs

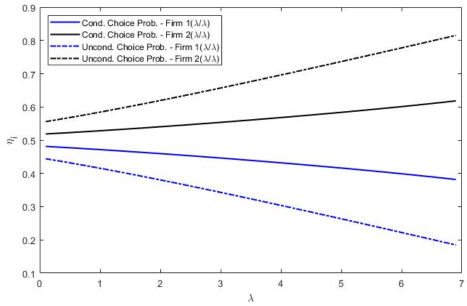
(a) Choice probabilities



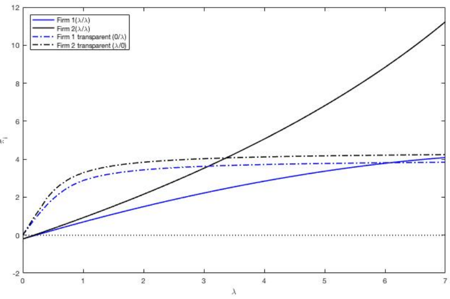
(b) Profits



(c) Choice probabilities, lower variance



(d) Profits, lower variance



Notes: The four parts of this figure illustrate conditional $\eta_i(\mathbf{k})$ and unconditional choice probabilities η_i^0 of the consumer and profits of firms for increasing obfuscation parameter (λ) and different obfuscation choices. We simulate asymmetric prior beliefs as an independent multinomial normal distribution with a mean of $\mu = (4.4, 4)$. Parts (a) and (b) of this figure present choice probabilities with a variance of $\sigma^2 = (2, 2)$. The blue line refers to the choice unconditional and conditional choice probabilities as well as the profit functions for firm 1 while the black line shows profits for firm 2. Parts (c) and (d) of the figure present results for a lower variance of prior beliefs, i.e., $\sigma^2 = (1.5, 1.5)$.

First, the prior beliefs may imply that consumers will not process any information about the new product (and thus will not buy it) if the new competitor obfuscates its price. The necessary and sufficient condition provided by Caplin et al. (2019) can be used as a simple test to determine whether the consumers would consider the new product or not. Besides a high expected price, this scenario may also arise if the consumer thinks the new product duplicates an existing one, i.e., yielding the same payoff, in contrast to the standard logit model (Caplin et al., 2019, MM15). In this scenario, the new competitor optimally chooses a transparent pricing scheme in the second stage to make at least some revenue. Alternatively, the new competitor may also not enter the market at all. In this sense, consumers' prior beliefs may create an additional source of a market entry barrier in a market with rational inattentive consumers.

Similar to the duopoly, we show that in a market with one transparent firm and two obfuscating firms, the transparent firm will again optimally choose the maximum price that will claim the entire market—the price cutoff level $\bar{p}_i$ (See Appendix D, Proposition 4).[15] Obfuscating firm(s) would incur a loss of

[15]This finding only holds under certain conditions that are always satisfied if we assume that, conditional on not choosing

$-\xi$ in this case, implying that a transparent pricing scheme is always the best reply for firms if there is one transparent firm in the market. Consequently, there always exists a subgame perfect equilibrium in which all firms do not obfuscate and choose to set a price of zero, $s_i = (\lambda_i = 0, p_i = 0) \quad \forall i \in N$. We are back in a Bertrand situation that yields zero equilibrium profits for all firms. Therefore, an obfuscation equilibrium ceases to exist if the prominence level of the new competitor is comparably low.

Second, the prior beliefs may imply that it is optimal for the consumers to process at least some information about all products if all firms obfuscate, $\eta_i^0 > 0, \forall i$. The derivation of the subgame perfect equilibria follows the same logic as in the duopoly case. For the existence of a pure strategy equilibrium, in which all firms choose to obfuscate, the profits in the mutual obfuscation case have to be higher than in the opposing cases for all firms. Otherwise, at least one of the firms would have an incentive to deviate, implying that no obfuscation equilibrium exists (see Proposition 3 for the equivalent condition in the duopoly case). In a market in which all firms obfuscate, the optimal pricing of the firm's strategy results in the following equilibrium profit for firm i (see Appendix D for the derivation):

$$\pi_i^* = \frac{\lambda}{(1 - \eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k}), \quad \forall i \in N$$

Compared to the duopoly case, a new competitor affects the optimal behavior of firms through two channels. On the one hand, as $\eta_i(\mathbf{k}) > 0 \, \forall \, i \in N$, a new competitor reduces aggregated market shares for existing firms. This decreases the prices and profits of existing firms through increased market competition. In turn, obfuscation becomes less attractive. On the other hand, a new market entry also changes the consumer's prior belief and thus alters the optimal information strategy of the consumer. As a result, the choice among existing alternatives may change. A single existing firm may benefit from a new market entry under rational inattention, in contrast to any random utility model (MM15). This information channel may also shift the equilibrium in the first stage, potentially in both directions, depending on whether the new information strategy favors the less prominent or more prominent firm.

The outcome depends on the exact shape of the consumer's prior belief, particularly on the correlation of the new competitor's price with the price of existing alternatives. Deriving the exact cutoff levels for different beliefs goes beyond this paper's scope, but consider the following stylized example, in which the likelihood of an obfuscation equilibrium may increase. There are two firms in the market with completely independent prices (according to the consumer's prior belief). One option has a small variance, and the other has thicker tails, implying a high probability that the firm's price is either very low or very high– a risky option. The option with the small variance seems more attractive to the consumer, such that obfuscation equilibria only exist for large values of λ. Now assume there is a new competitor, which also appears risky to the consumer. Prices of the new competitor are perceived to be negatively correlated with prices of the less prominent firm but independent from the prices of the more prominent firm. This

the transparent option, the other products are a priori homogeneous. Or, less restrictively, if increasing the "prominence," i.e., unconditional choice probabilities, of firm i decreases the prominence of both other firms.

negative perceived correlation increases the incentive for the consumer to investigate whether one of the risky options is very cheap If, in addition, unconditional choice probabilities and resulting profits are more balanced after market entry, the likelihood of an obfuscation equilibrium may increase, even if aggregated demand in the obfuscation case decreases for existing firms.

In addition, a new competitor may increase the incentive to investigate prices at all. The higher the attractiveness for the consumer to investigate the obfuscated alternatives, the lower $\bar{p}_i$ is, and thus the incentive for one particular firm to deviate from the obfuscation equilibrium is lower. As a consequence, the likelihood of an obfuscation equilibrium may increase.

Following the arguments above, it is impossible to give a general answer on the effects of the number of firms on firms' obfuscation choices. However, in the special case of symmetric prior beliefs, it is straightforward to see that obfuscation equilibria cease to exist if the number of firms is high. Note that in the case of symmetric prior beliefs, the unconditional prior beliefs are equal to $\eta_i^0 = \frac{1}{N}$ and the second stage in the joint obfuscation case reduces to the standard logit oligopoly Anderson et al. (1992). Optimal prices in the joint obfuscation case are equal to:

$$p^* = \lambda \frac{N}{N-1}.$$

Associated profits are given by:

$$\pi^* = \frac{\lambda}{N-1} - \xi$$

Consequently, the profits in the joint obfuscation case decrease in the number of firms and converge to $-\xi$ for $N \to \infty$. So, given that deviating to a transparent pricing scheme will result in non-negative profits, obfuscation equilibria cease to exist if the number of firms is large enough (see Proposition 3).

5 Conclusion

This paper examines firms' equilibrium behavior with rational inattentive consumers in a duopoly. Before firms compete on the price of a homogeneous product, firms decide whether to obfuscate the prices or disclose all relevant information. In our motivating example of the mobile subscription market, mobile operators may obfuscate prices by using add-on pricing schemes with variable fees, including diverging charges for different operators and call times, non-linear prices for data usage, or varying roaming fees. Compared to a fixed-fee pricing scheme, assessing the final prices is more complicated and requires costly effort by consumers to get a reasonable estimate of future consumption and understand all contractual details.

The presence of rational inattentive consumers may make it rational for firms to obfuscate prices in equilibrium. However, mutual obfuscation equilibria with high prices and profits only exist if consumers' information costs of learning about obfuscated prices are high enough. If information costs are low, the rent from obfuscation is small, and firms have an incentive to deviate from obfuscation in equilibrium. Obfuscation is not a mutually best reply anymore, and the unique equilibrium will be one with transparent prices and equilibrium profits of zero. In contrast to most related studies, unilateral obfuscation is not profitable in equilibrium. Furthermore, we show that firms' mutual obfuscation in equilibrium ceases to exist if, according to the prior beliefs, the consumers expect one offer to be superior to other offers. The asymmetry in prior beliefs is novel to the obfuscation literature and can capture the concept of firms' "prominence" in a rational inattention framework.

Besides attentional costs for consumers, obfuscation has negative welfare implications as it implies decreased competition among firms. For these reasons, policymakers may want to limit obfuscation. According to our findings, this can be achieved by decreasing consumers' information costs. Potential policy measures include fostering the creation of neutral product comparison portals, ensuring consumers' access to digital services, and increasing comparability between products, such as forbidding some price dimensions that are difficult to access or standardizing product descriptions. Furthermore, new and unknown competitors may decrease obfuscation behavior in equilibrium because they are likely to increase the relative difference in prominence between competing firms. Therefore, lowering market entry barriers may further positively affect social welfare.

Our study has several limitations and implications that open several avenues for future research. First, firms' obfuscation decisions in the first stage are binary. This assumption is crucial for our results and allows for tractable results. While it provides a useful simplification for markets, as in our motivating example, where a simple (flat) and complex (add-on) price frame prevails, it may be worthwhile to allow for more flexibility in this respect to assess questions about the fragmentation of other markets and different scopes and degrees of obfuscation. Huettner et al. (2019) may provide a suitable framework for the consumer side that can be applied to study the market outcomes under continuous obfuscation choices.

Second, prior beliefs in our model are unrestricted and exogenous, which relaxes the rational expectations assumption and allows for a unifying, simple and general framework. However, it also neglects equilibrium feedback effects on consumers' beliefs and behavior, such as inference from firms' observed obfuscation choices. It further implies that our model is agnostic about the mechanisms behind the prominence of firms and the source of uncertainty in our model. Putting more structure on prior beliefs may thus help to learn more about feedback mechanisms and their implications for firms' optimal obfuscation and pricing behavior. Related to the prior beliefs, one could also discuss obfuscation that, in addition to the information costs of consumers, also affects the shape of consumers' prior beliefs.

Third, we consider a static model. Extending our framework to a dynamic setup would allow us to study the effects of memory and learning under rational inattention on market equilibria and optimal obfuscation behavior over time. Steiner et al. (2017) and Maćkowiak et al. (2018) study rational inattention in a dynamic framework. While dynamic applications of rational inattention in the industrial organization literature are rare, this approach seems particularly promising in markets characterized by a dynamic environment.

Fourth, the effect of digitalization on economic activity is an empirical question that has received much attention from scholars in recent years (Goldfarb and Tucker, 2019). Combining the insights from rational inattention theory with empirical work on these issues can help to identify better the decisive role of information in the far-reaching digital transformation process that reshapes many disciplines, including economics.

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Appendix

A Application: The Swedish Mobile Subscription Market

This section provides a descriptive analysis of the Swedish mobile subscription market to exemplify our motivation. We document stylized facts about the development of mobile subscriptions.

Data

We use yearly market share data for the Swedish mobile subscription market between 2011 and 2018 provided by the Swedish Post and Telecom Authority. In detail, we observe yearly revenues of firms fragmented by type of subscription (i.e., prepaid and postpaid) and fees (i.e., variable and fixed fees). We adjust revenues for inflation using general yearly consumer price indices. Additionally, we use the consumer price index (CPI) for the product group of telephone services and equipment from Statistics Sweden.

Prices and Subscriptions

We start by evaluating prices in the subscription market. Even though we do not observe prices in our data set, we use several measures that allow us to infer average price movements. Figure A.1a shows the development of the CPI for telecommunication and telecommunication equipment between 2011 and 2015. In this time period, the yearly CPI dropped by 10%. In Figure A.2 of Appendix A we document that the producer prices, which do not include equipment expenses, show the same trend. Between 2014 and 2020, producer prices decreased by 13%. The same holds true for average revenues per subscription in Figure A.3, which also decreased over the last two decades. Furthermore, decreasing prices are not a unique phenomenon of the Swedish mobile subscription market. Indeed, average revenues per user in the mobile subscription market decreased in most European countries (Commission, 2019), which leads us to our first stylized fact.

Stylized Fact 1. In recent years, prices in the mobile subscription market decreased.

Figure A.1b illustrates aggregated yearly revenues between 2011 and 2017. Total revenues increase slightly. Looking at yearly revenues divided into fixed and variable fees subscriptions, we see that this increase can be attributed to a strong increase in fixed subscription fees. In comparison, the revenues that are due to variable fees significantly decreased. In 2011 54% of the yearly revenues could be attributed to fixed fees, while in 2017 fixed fees were responsible for 83% of the revenues. We observe a similar heterogeneous development in the number of subscriptions in Figure A.1c. Overall, the market size of the mobile market increased slightly. Between 2011 and 2017, the total number of subscriptions increased from 13.4 to 14.4 million. As we saw above, the increasing number of subscriptions increases the aggregate revenues, while average revenues per subscriptions decrease in accordance with stylized fact 1 (see Figure

A.3). If we divide the number of subscriptions into postpaid and prepaid subscriptions between 2011 and 2017, we see an increase in the share of postpaid subscriptions from 66% to 76%, while the share of prepaid subscriptions decreased. This is due to the fact that fixed fee subscriptions are usually postpaid, while variable fees are indicative for prepaid subscriptions. Looking at the characteristics of the subscriptions, we conclude:

Stylized Fact 2. Fixed-fee and postpaid subscriptions became more common, replacing variable-fee and prepaid subscriptions.

Intuitively, both the increase in fixed fees and the increase in postpaid contracts are in line with the reasoning that consumers purchase easier contracts. The complexity in pricing schemes of mobile subscriptions has decreased. At the same time, the average price decreased, while the number of products increased. We argue that the descriptive example of the Swedish market is representative of mobile subscription markets in general. Historically, we observed markets with variable pricing structures where it has been difficult to assess mobile services' final price per year or month. In recent years, we observe not only reduced prices but also less complex pricing schemes.

Competition

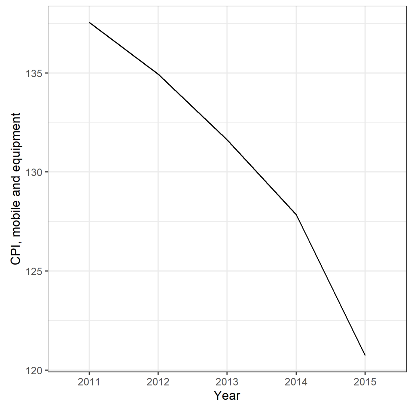
Additionally, we observe an increase in competition in the Swedish mobile subscription market. The source of competition in the market is manifold. On the one hand, the distribution of 3G licenses in 2000 led to the entry of a new competitor that gained market share relatively fast (OECD, 2015). On the other hand, network-sharing agreements that allow easier entry are common and frequently used (OECD, 2015). Figure A.1d shows the market shares of the four biggest operators. After the entry of a fourth competitor in 2010, market shares of all companies remained largely unchanged. However, we observe slight decreases in market shares of the two largest firms (around 5 percentage points each) while smaller firms increased their market shares.

Producer Price Index and Average Revenues

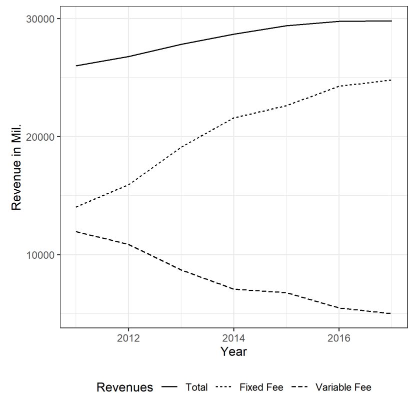
Our main empirical analysis argues that the decrease in the consumer price index in telecommunication services and equipment is a clear indicator of decreasing prices. Within this section, we show that the result is similar when evaluating the producer price index or average revenues. Figure A.2 shows the yearly producer price index for telecommunication services in Sweden between 2014 and 2020. Figure A.3 presents monthly average revenues in the mobile subscription market between 2000 and 2018. Similar to the results for the consumer price index, the producer price index and the average revenues decrease.

Figure A.1: The Swedish Mobile Subscription Market

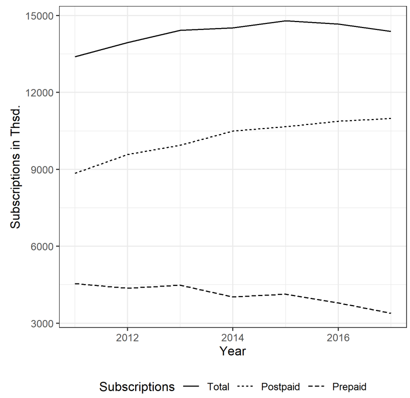
(a) Consumer Price Index, Telecommunication



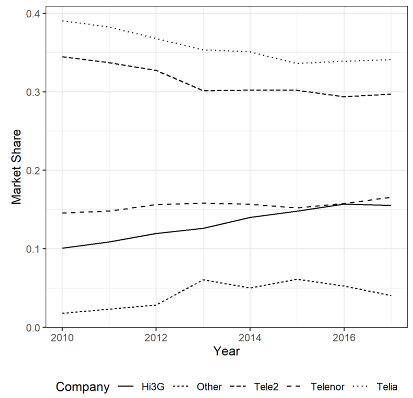
(b) Revenues



(c) Subscriptions



(d) Market Shares



Notes: The four parts of this figure illustrate key stylized facts of the Swedish mobile subscription market: (a) the yearly consumer price index for telecommunication and telecommunication equipment between 2011 and 2015; (b) aggregated revenue, divided into fixed and variable fees and adjusted for inflation, in 2011 prices; (c) the number of mobile subscriptions in the Swedish mobile subscription market between 2011 and 2017, divided into postpaid and prepaid subscriptions; and (d) the four biggest competitors' market shares between 2011 and 2017.

B Solving the Consumer Maximization Problem

Solving the consumer maximization problem, we mainly follow MM15. Formally, in the non-trivial cases the consumer chooses an information strategy to solve the following maximization problem:

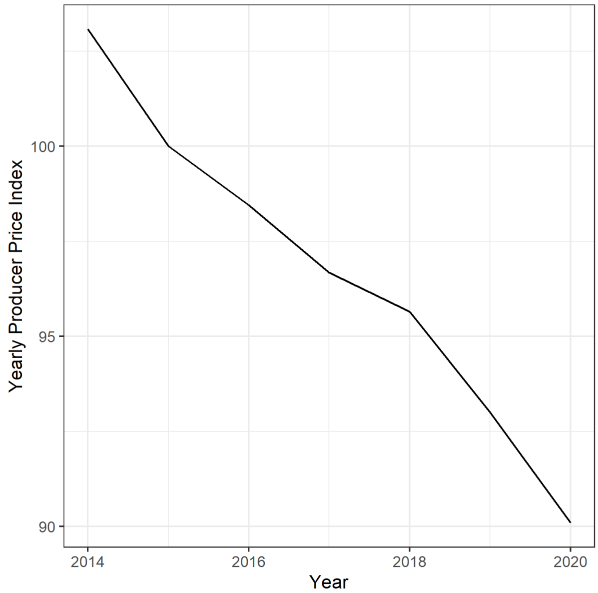
$$\max_{F\in\Delta(\mathbb{R}^{2N})} \int_{\mathbf{k}}\int_{\mathbf{z}} E(k_{a(F(\mathbf{k}|\mathbf{z}))})F(d\mathbf{z}|\mathbf{k})G_0(d\mathbf{k}) - \hat{c}(F)$$

$$\text{s.t. } \int_z F(d\mathbf{z},\mathbf{k}) = G_0(\mathbf{k})$$

Let Z_i further be the set of signals $\mathbf{z}$ that result in action strategy a:

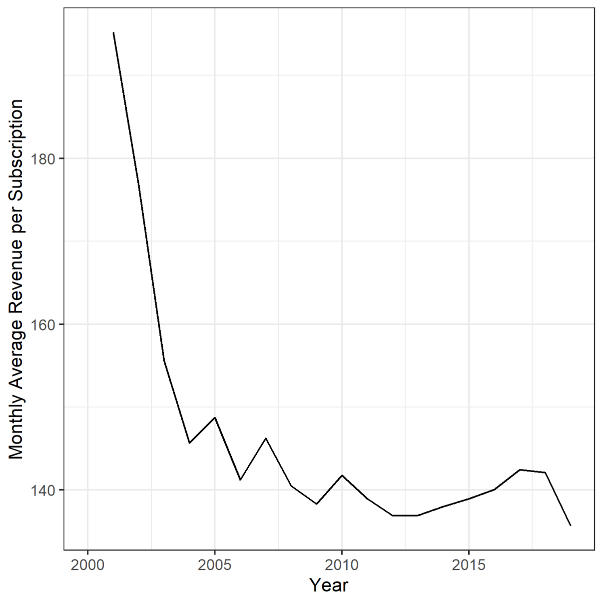
$$Z_i = \{\mathbf{z}\in\mathbb{R}^N : a(F(\mathbf{k}|\mathbf{z})) = i\},$$

Figure A.2: Producer Price Index



Notes: The figure presents the yearly producer price index for telecommunication services in Sweden between 2014 and 2020. The baseline of the index is in the year 2015.

Figure A.3: Monthly Average Revenue



Notes: The figure presents the monthly average revenue per subscriber in the Swedish mobile subscription market between 2000 and 2015. Average revenues are adjusted for inflation and in prices of 2000.

where N is the number of options available to the consumer ($N = 2$ in our base case). Building upon this, we define the conditional probability of selecting action i depending on state $\mathbf{k}$:

$$\eta_i(\mathbf{k}) \equiv \int_{z \in Z_i} F(d\mathbf{z}|\mathbf{k})$$

The unconditional choice probabilities before observing the costly signal but after processing all costless information is given by the following:[16]

$$\eta_i^0 \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) G_0(d\mathbf{k}) \tag{6}$$

It can be shown that, due to convex information costs, one action strategy is selected in at most one posterior (MM15). This implies that each action strategy is associated with a particular signal, which in turn allows us to rewrite the maximization problem of the consumer in terms of choice probabilities, which facilitates the derivation of the respective demand. Without loss of generality we restrict the derivation of the conditional demand to two options, i.e., two products the consumer can choose from. Using the definition of Shannon entropy, our maximization problem then reads:

$$\max_{\eta \in \{\eta_i(\mathbf{k})\}_{i=1}^2} \sum_{i=1}^{2} \int_{\mathbf{k}} k_i \eta_i(\mathbf{k}) G_0(d\mathbf{k})) - \lambda(-\sum_{i=1}^{2} \eta_i^0 log \eta_i^0 + \int_{\mathbf{k}} \sum_{i=1}^{2} \eta_i(\mathbf{k}) log \eta_i(\mathbf{k})) G_0(d\mathbf{k}) \tag{7}$$

such that

$$\eta_1(\mathbf{k}) + \eta_2(\mathbf{k}) = 1 \tag{8}$$

$$\forall i : \quad \eta_i(\mathbf{k}) \geq 0, \tag{9}$$

where η is the collection of conditional probabilities $\{\eta_i(\mathbf{k})\}_{i=1}^2$. The maximization problem is applicable to all cases in which at least one firm chooses to obfuscate its prices. However, the cases differ with respect to the relevant (conditional) prior beliefs. We denote the prior belief conditional on all costless information $\tilde{k}$ by: $\Omega_0(\mathbf{k}) = G_0(\mathbf{k}|\tilde{k})$.

Both Firms Obfuscate

In the case in which both firms obfuscate ($\lambda_1 = \lambda_2 = \lambda$), the consumer chooses an information strategy based on $G_0(\mathbf{k})$ and homogeneous information costs across both products. Choosing an informative signal, she will thus consider the joint distribution about both prices. This makes our problem directly applicable to the framework studied by MM15.

[16] Again, what is relevant to our problem is not the unconditional choice probability before any information $G(\cdot)$ is processed, but the unconditional choice probability after all costless information is processed.

Solving the corresponding Lagrangian, where $\zeta(\mathbf{k})$ signifies the Lagrange multipliers on (4) and $\tau_i(\mathbf{k})$ are Lagrange multipliers on (5), gives us the following first-order conditions (assuming an interior solution $\eta_i^0 > 0$):

$$k_i - \lambda\left(-log(\eta_i^0) - 1 + log(\eta_i(\mathbf{k}) + 1\right) + \tau_i(\mathbf{k}) - \zeta(\mathbf{k}) = 0$$

MM15 show that if $\eta_i^0 > 0$ and $k_i > -\infty$, it has to hold that $\eta_i(\mathbf{k}) > 0$, which in turn implies that the Lagrange multiplier on (6) is zero, $\tau_i(\mathbf{k}) = 0$. Taking the exponential of both sides and rearranging the first-order condition results in:

$$\eta_i(\mathbf{k}) = \eta_i^0 e^{(k_i - \zeta(\mathbf{k}))/\lambda} \tag{10}$$

Plugging (7) into (5) gives us:

$$e^{\zeta(\mathbf{k})/\lambda} = \sum_{i=1}^{2} \eta_i^0 e^{k_i/\lambda},$$

which in turn can be plugged back into (7) to arrive at our demand function (1) for $0 < \eta_i^0 < 1$. Note that η_i^0 is not only determined by the prior beliefs but is rather a result of the maximization problem itself. We plug the conditional choice probabilities (1) into the definition of the unconditional choice probabilities (3) to arrive at the normalization conditions that allow us to numerically solve for the unconditional choice probabilities:

$$\int_{\mathbf{k}} \left(\frac{e^{k_i/\lambda}}{\sum_{j=0}^{2} \eta_j^0 e^{k_j/\lambda}} \right) G_0(d\mathbf{k}) = 1, \quad \forall i \eta_i^0 > 0$$

If $\exists \eta_i^0 = 0$, then $\eta_i(\mathbf{k}) = 0$. This implies that the consumer does not process any information about i and, in the duopoly case, always buys the product of the competitor. Caplin et al. (2019) provide the necessary and sufficient boundary conditions for this case that can be applied to our simple two-product setting.

Only One Firm Obfuscates

The logic of calculating the choice probabilities if only one firm obfuscates is similar. Note that we assume here without loss of generality that $\lambda_1 = 0$ and $\lambda_2 = \lambda$. However, when deciding how much information to process (about firm 2's price), the consumer knows the price of product 1. This has some implications for the relevant conditional prior belief that is given by $\Omega_0(k_2) = G_0(k_2|k_1)$, implying that the uncertainty is one-dimensional, considering only product 2's price. Note that if the price of product 1 is low enough compared to the expected price of firm 2, the consumer will not process any information about product 2 and will just buy product 1. Similarly, if the expected price of product 2 is lower than p_1, the consumer will pay no attention and will just buy (uncertain) product 2. This case's logic is similar to that of the case in which the consumer can decide to buy a product/enter the market or choose an outside option with a certain outcome. A similar setting is discussed in MM15 or Boyacı and Akçay (2017) for a binary state variable and corresponding prior belief that follows a Bernoulli distribution.

Plugging constraint (5) into the maximization problem above, noting that k_1 is perfectly known by the consumer, and assuming $0 < \eta_2^0 < 1$, the Lagrangian for the maximization problem of the consumer in the case where only one firm obfuscates can be rewritten to:

$$\max_{\eta_2(\mathbf{k})} \int_{k_2} k_2 \eta_2(\mathbf{k}) \Omega_0(dk_2) + k_1 \int_{k_2} (1 - \eta_2(\mathbf{k})) \Omega_0(dk_2)$$
$$-\lambda \left(-\eta_2^0 log \eta_2^0 - (1 - \eta_2^0) log(1 - \eta_2^0) + \int_{k_2} (\eta_2(\mathbf{k}) log \eta_2(\mathbf{k}) + (1 - \eta_2(\mathbf{k})) log(1 - \eta_2(\mathbf{k}))) \Omega_0(dk_2) \right)$$

Differentiating with respect to $\eta_2(\mathbf{k})$ and noting that $\eta_2(\mathbf{k}) > 0$ almost surely if $\eta_2^0 > 0$ and $k_2 > -\infty$ (see MM15) gives us the following first-order condition:[17]

$$(k_2 - k_1) - \lambda \left(-log(\eta_2^0) - 1 + log(1 - \eta_2^0) + 1 + log(\eta_2(\mathbf{k})) + 1 - log(1 - \eta_2(\mathbf{k})) - 1 \right) = 0$$

Combining the terms and taking the exponential on both sides gives us:

$$e^{(k_2 - k_1)/\lambda} = \frac{1 - \eta_2^0}{\eta_2^0} \frac{\eta_2(\mathbf{k})}{1 - \eta_2(\mathbf{k})}$$

Rearranging and adding 1 (respectively $\frac{(1-\eta_2^0)}{(1-\eta_2^0)}$ and $\frac{(1-\eta_2^0)e^{k_1/\lambda}}{(1-\eta_2^0)e^{k_1/\lambda}}$) on both sides gives us:

$$\frac{1}{1 - \eta_2(\mathbf{k})} = \frac{(1 - \eta_2^0) e^{k_1/\lambda} + \eta_2^0 e^{k_2/\lambda}}{(1 - \eta_2^0) e^{k_1/\lambda}}$$

Using the definition of k_i gives us our final demand:

$$\eta_2(\mathbf{k}) = \frac{\eta_2^0 e^{k_2/\lambda}}{(1 - \eta_2^1) e^{k_1/\lambda} + \eta_2^0 e^{k_2/\lambda}}$$

$$\eta_1(\mathbf{k}) = 1 - \eta_2(\mathbf{k})$$

The normalization condition gives us our unconditional choice probabilities:

$$\int_{k_2} \left(\frac{e^{k_2/\lambda}}{\eta_2^1 e^{k_2/\lambda} + (1 - \eta_2^1) e^{k_1/\lambda}} \right) \Omega_1(dk_2) = 1$$

:

[17] Note that $\eta_i^l \equiv \int_{\mathbf{k}} \eta_i(\mathbf{k}) G_l(d\mathbf{k})$.

C Simultaneous Move Game

The following section shows that a simultaneous move game where firms set prices simultaneously to obfuscation results in the same pure strategy equilibria. We follow a two-step procedure to prove this claim. First, we show that no opposing equilibria exist. Second, we show that the equilibria in the simultaneous version of the game are equivalent to the sequential move game.

The general action space of the game remains as before. However, firms decide on obfuscation $\lambda_i \in \{0, \lambda\}$ and prices $p_i \geq 0$ simultaneously. Afterward, the rational inattentive consumers make their purchase decision. As a result, we search for a subgame perfect equilibrium that coincides with a Nash equilibrium. We search for mutual best replies of firms given the demand that follows the rational inattention framework.

First, note that any equilibrium where one firm chooses to obfuscate ($\lambda_1 = \lambda$) while the other does not obfuscate ($\lambda_1 = 0$) does not exist, independent of the size of λ or prior beliefs. Consider first the non-obfuscating firm. Following proposition 2, it still holds that the optimal price for the non-obfuscating firm ($\lambda_i = 0$) is $\bar{p}_i$, a price that allows the firm to claim the entire market given prior beliefs and obfuscation costs. This result is independent of the obfuscating firm's price. As the obfuscating firm has a market share of zero and obfuscation costs, obfuscation with any possible price can never be the best reply to a non-obfuscating firm with a price of $\bar{p}_i$. The obfuscating firm would deviate from transparency. Thus, an opposing equilibrium does not exist.

Second, we show that the transparency and obfuscation equilibria are the same as for the sequential move game. First, consider the transparency equilibrium with no obfuscation $\lambda_i = 0$ and $p_i = 0 \; \forall i \in N$. The proof that the transparency with a zero price is a mutual best reply is identical to the one in Proposition 3.i. Additionally, deviating from transparency to obfuscation would result in zero revenue, as the opponent with a zero price would capture the entire market independent of the obfuscating price. As obfuscation comes with a cost, obfuscation is never optimal. Thus, a transparency equilibrium is the same for a simultaneous move game.

Now we consider the obfuscation equilibrium. From Proposition 1, we already know that given the choice of obfuscation, the unique price that satisfies a mutual best reply is $p_i = \dfrac{\lambda}{(1 - \eta_i^*(\mathbf{k}))}$. It remains to show that deviating to transparency is never optimal. Following Proposition 2, the optimal deviation to transparency would involve a price $\bar{p}_i$ as it maximizes the firm's profits under transparency. Any other price of the transparent firm would lead to lower profits and, therefore, weaker requirements for the obfuscation equilibrium. Therefore deviation to transparency is non-optimal under condition (4) of Proposition 3.iii. Thus, we are in the same situation as in the sequential game. We observe an obfuscation equilibrium in the simultaneous game for the same values of prior beliefs $G_0(\cdot)$ and information costs λ as in the sequential move game.

D Derivation of Subgame Perfect Equilibria with N Firms

In a market in which all firms obfuscate, the optimal information strategy results in a generalized multinomial logit demand function for all products:[18]

$$\eta_i(\mathbf{k}) = \frac{\eta_i^0 e^{(q-p_i)/\lambda}}{\sum_{j=1}^{N} \eta_j^0 e^{(q-p_j)/\lambda}}, \quad \forall i \in N$$

Following Anderson et al. (1992) and as in the duopoly, the subgame perfect equilibria are uniquely determined by following system of equations:

$$p_i^* = c + \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}, \quad \forall i \in N$$

with equilibrium profits of

$$\pi_i^* = \frac{\lambda}{(1-\eta_i^*(\mathbf{k}))}\eta_i^*(\mathbf{k}), \quad \forall i \in N$$

This proposition and proof are the generalization of Proposition 2 to a market with N competing firms. While the equilibrium conditions in the second stage for the cases with identical obfuscation strategies also apply more generally to the N-firms case, the outcome of cases when firms choose diverging obfuscation strategies becomes very involved and dependent on the belief structure if firms are heterogeneous according to consumers' prior beliefs. For this reason, we focus on $N = 3$ here. Without loss of generality, we further assume that firm 1 is using the transparent pricing scheme.

Proposition 4. *With two obfuscating firms ($i = 2, 3$) and one transparent firm ($i = 1$), the optimal price of the transparent firm ($\lambda_1 = 0$) is equal to $\bar{p}_1$, where $\bar{p}_1 = \max p_1$ such that $\eta_1^0 = \eta_1(\boldsymbol{k}) = 1$.*

Proof. As in Proposition 2, note that the transparent firm will not set a price below $\bar{p_1}$, as $\frac{\partial \pi_1}{\partial p_1} = 1 > 0$ if $p_i < \bar{p_1}$, $\eta_1(\mathbf{k}) = 1$.

All firms will face a multinomial logit demand if $p_1 \geq \bar{p_1}$ (see above). The maximized profit function for a given unconditional choice probability is equal to $\pi_1^*(\eta_1^0) = \frac{\lambda}{(1-\eta_1^*(\mathbf{k}))}\eta_1^*(\mathbf{k})$.

Plugging in the corresponding demand function, the profit function of firm 1 can be rewritten to:

$$\pi_1^*(\eta_1^0) = \lambda \frac{\eta_1^0 e^{(q-p_1^*)/\lambda}}{\eta_2^0 e^{(q-p_2^*)/\lambda} + \eta_3^0 e^{(q-p_3^*)/\lambda}} = \lambda \frac{\eta_1^0 e^{-p_1^*/\lambda}}{\eta_2^0 e^{-p_2^*/\lambda} + \eta_3^0 e^{-p_3^*/\lambda}}$$

If we assume that, conditional on not choosing the transparent option (option 1), the other products are a priori homogeneous, implying equal unconditional choice probabilities (η_{-i}^0) and optimal prices (see, e.g., Matějka and McKay, 2012). Then, the unconditional choice probabilities can be rewritten as $\eta_2^0 = \eta_3^0 = \frac{(1-\eta_1^0)}{2}$. This simplifies the maximized profit function to:

[18] See Appendix A for the derivation.

$$\pi_1^*(\eta_i^0) = \lambda \frac{\eta_1^0 e^{(q-p_1^*)/\lambda}}{\frac{(1-\eta_1^0)}{2}\left(e^{(q-p_2^*)/\lambda} + e^{(q-p_3^*)/\lambda}\right)} = 2\lambda \frac{\eta_i^0}{(1-\eta_i^0)} \frac{e^{(-p_1^*)/\lambda}}{\left(e^{(-p_2^*)/\lambda} + e^{(-p_3^*)/\lambda}\right)}$$

Applying the envelope theorem, differentiating with respect to η_1^0 equals:

$$\frac{\partial \pi_i^*}{\partial \eta_i^0} = \frac{2\lambda}{(1-\eta_1^0)^2} \frac{e^{(-p_1^*)/\lambda}}{\left(e^{(-p_2^*)/\lambda} + e^{(-p_3^*)/\lambda}\right)} > 0,$$

which is strictly larger than zero if $\eta_1^0 < 1$.

If the options are not a priori homogeneous, the derivative of the maximized profit function with respect to η_1^0 is given by:

$$\frac{\partial \pi_1^*}{\partial \eta_1^0} = \lambda \frac{e^{(-p_1^*)/\lambda}\left((\eta_2^0 e^{(-p_2^*)/\lambda} + \eta_3^0 e^{(-p_3^*)/\lambda}) - \eta_1^0 \left(\frac{\partial \eta_2^0}{\partial \eta_1^0} e^{(-p_2^*)/\lambda} + \frac{\partial \eta_3^0}{\partial \eta_1^0} e^{(-p_3^*)/\lambda} \right) \right)}{(\eta_2^0 e^{(-p_2^*)/\lambda} + \eta_3^0 e^{(-p_3^*)/\lambda})^2}$$

This is always larger than zero if $\left(\frac{\partial \eta_2^0}{\partial \eta_1^0} e^{(-p_2^*)/\lambda} + \frac{\partial \eta_3^0}{\partial \eta_1^0} e^{(-p_3^*)/\lambda} \right) \leq 0$, which always holds if we assume that increasing the "prominence" i.e., unconditional choice probabilities, of firm 1 decreases the prominence of both other firms.[19]

□

[19]This assumption can further be relaxed by considering the entire numerator.